Supplementary information
for
High-speed readout for direct light orbital angular momentum photodetector via photoelastic modulation

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S1. OPGE response of the MLG device

The derivation of the OPGE response has been fully described in Ref. [1]. In general, the OPGE response arises from the electric quadrupole and magnetic dipole effects induced by the light phase gradient, corresponding to the response term $\boldsymbol{J}_{qp}$. It can be divided into four terms according to its dependence on the SAM and OAM:

$$\boldsymbol{J}_{qp}(\rho,\theta,z) = m\cdot\sigma_i\,\boldsymbol{J}_{(1)}(\rho,\theta,z) + m\,\boldsymbol{J}_{(2)}(\rho,\theta,z) + \sigma_i\,\boldsymbol{J}_{(3)}(\rho,\theta,z) + \boldsymbol{J}_{(4)}(\rho,\theta,z) \tag{S1.1}$$

where the first term $m\cdot\sigma_i\,\boldsymbol{J}_{(1)}$ is proportional to the product of SAM (σ_i) and OAM (m) and changes its sign when the SAM switches from +1 to -1. Since the SAM is related to the left/right circular polarization and is tunable in circular photogalvanic effect (CPGE) detection, it can be extracted via CPGE measurement, with the extracted component $m\cdot\boldsymbol{J}_{(1)}$ proportional to the OAM order. If we ensure that the total power and ring radius of the OAM beam remain unchanged for different m values, the measured CPGE response from $m\cdot\boldsymbol{J}_{(1)}$ has a quantized magnitude on the OAM order m, which enables detection of the OAM order. The third term $\sigma_i\,\boldsymbol{J}_{(3)}$ changes its sign when the SAM order σ_i switches from +1 to -1, but it shows no dependence on the OAM order m and gives a background signal in the CPGE measurement. The second term $m\,\boldsymbol{J}_{(2)}$ and the fourth term $\boldsymbol{J}_{(4)}$ have no circular polarization dependence and are removed when the circular polarization-dependent component is extracted from CPGE measurements.

Here, we focused on the first term $m\cdot\sigma_i\,\boldsymbol{J}_{(1)}$, which is used for OAM detection. For the multilayer graphene used in this work, symmetry determines that the first term $m\cdot\sigma_i\,\boldsymbol{J}_{(1)}$ has only a radial component with the following expression:

$$m\cdot\sigma_i\,\boldsymbol{J}_{(1)}^{\rho} = m\cdot\frac{4E_0^2\left|u_{p,m}(\rho,z)\right|^2}{\rho}\frac{\sigma_i}{1+|\sigma|^2}\left(S_i^{xxyy}-S_i^{xyxy}\right) \tag{S1.2}$$

where E_0 is the amplitude of the light field, $u_{p,m}(\rho,z)$ is the normalized LG mode profile, and $\sigma = \sigma_r + i\sigma_i = \tilde{E}_y/\tilde{E}_x$ is the ratio of the complex amplitudes of the light field in two perpendicular polarization directions and describes the arbitrary polarization state of the OAM beam. The specific expression of $u_{p,m}(\rho,z)$ is given in Ref. [1].

For both mechanical and PEM modulation, the polarization undergoes one periodic switch between left circular ($\sigma_i = +1$) and right circular ($\sigma_i = -1$) polarizations in one operation cycle. However, the specific evolution of the polarization sequence (the variation in σ) is different, which can lead to differences in OPGE signal extraction. Moreover, corresponding to the different polarization modulation schemes, the CPGE extraction approaches are also different for mechanical and PEM modulations, which leads to different constant coefficients for the extracted CPGE response. In Section 2-3, we provide the derivation of the specific analytic expression of the modulated OPGE response and the extraction of the CPGE component to show the differences between these two different polarization modulation schemes.

S2. Modulated OPGE response for different polarization modulation schemes

To obtain the specific expression of the modulated OPGE response, we derive the expressions of the modulated polarization state, denoted by the complex $\sigma = E_y/E_x$, for mechanical and PEM modulation. For mechanical modulation, a quarter wave plate (QWP) is placed after a linear polarizer with the polarization direction along the x-direction and rotated to modulate the polarization states. When the quarter-wave plate is rotated relative to the polarizer by angle θ, the expression of the light field after the QWP is given by:

$$\tilde{E}_x = E_{p,m}(\vec{\boldsymbol{r}})(\cos^2\theta + i sin^2\theta),\quad \tilde{E}_y = E_{p,m}(\vec{\boldsymbol{r}})(1-i)\cos\theta\sin\theta \tag{S2.1}$$

and the expression of σ is given by:

$$\sigma = E_y/E_x = \frac{(1-i)\cos\theta\sin\theta}{\cos^2\theta + isin^2\theta} = \sigma_r + i\sigma_i \tag{S2.2a}$$

$$\sigma_r = \frac{\cos\theta\sin\theta\,(\cos^2\theta - \sin^2\theta)}{\cos^4\theta + sin^4\theta}, \sigma_i = -\frac{\cos\theta\sin\theta}{\cos^4\theta + sin^4\theta}, \quad \frac{1}{1+|\sigma|^2} = \cos^4\theta + sin^4\theta \tag{S2.2b}$$

If we consider that the QWP is continuously rotated with frequency f and $\theta = 2\pi ft$, the expression of $m \cdot \sigma_i \boldsymbol{J}_{(1)}^{\rho}$ is given by:

$$m \cdot \sigma_i \boldsymbol{J}_{(1)}^{\rho} = -m \cdot \frac{2E_0^2\left|u_{p,m}(\rho,z)\right|^2}{\rho}\left(S_i^{xxyy} - S_i^{xyxy}\right)\sin 4\pi ft = -m \cdot C(\vec{\boldsymbol{r}})\sin 4\pi ft \tag{S2.3}$$

where $C(\vec{\boldsymbol{r}}) = \frac{2E_0^2\left|u_{p,m}(\rho,z)\right|^2}{\rho}\left(S_i^{xxyy} - S_i^{xyxy}\right)$ is the coefficient that is determined by the light field distribution and the rank-4 conductivity tensors of the detection material. Experimentally, if we extract the 180°-periodicity component of the photocurrent response (CPGE component J_C), the extracted CPGE response corresponds to $-m \cdot C(\vec{\boldsymbol{r}})$.

In photocurrent measurements, the detected photocurrent can be expressed as the integration of the current density. When the U-shaped electrodes surround a region $S = [R_1, R_2][0, \pi]$ in polar coordinates, the collected CPGE response is given by:

$$I_C^{\rho} = \int_S -m \cdot C(\vec{\boldsymbol{r}})d\boldsymbol{r} = -m \cdot \int_S C(\vec{\boldsymbol{r}})d\boldsymbol{r} \tag{S2.4}$$

If we keep the rings of the LG beams embedded inside the region S by adjusting the focal distance, $\int_S C(\vec{\boldsymbol{r}})d\boldsymbol{r}$ can be approximated by:

$$\int_S C(\vec{\boldsymbol{r}})d\boldsymbol{r} = \int_0^{\pi} d\theta \int_{R_1}^{R_2} \frac{2E_0^2\left|u_{p,m}(\rho,z)\right|^2}{\rho}\left(S_i^{xxyy} - S_i^{xyxy}\right)\rho d\rho = 2\pi b_1\left(S_i^{xxyy} - S_i^{xyxy}\right)$$
$$\approx \frac{2w_0^2E_0^2}{(R_1+R_2)}\left(S_i^{xxyy} - S_i^{xyxy}\right) \tag{S2.5}$$

where $b_1 = \int_{R_1}^{R_2}\left|u_{p,m}(\rho,z)\right|^2 d\rho \approx \frac{w_0^2}{\pi(R_1+R_2)}$, $w_0^2 = \int_{R_1}^{R_2}\left|u_{p,m}(\rho,z)\right|^2 d\boldsymbol{r}$ remains unchanged for different OAM orders m. Therefore, $\int_S C(\vec{\boldsymbol{r}})d\boldsymbol{r}$ remains approximately unchanged for different OAM orders, and the collected CPGE response I_C^{ρ} is proportional to the OAM order m.

For PEM modulation, a polarizer is positioned before the PEM at a 45° angle between the principal axes to produce a 45° linearly polarized beam. A periodic phase retardation $\delta = \delta_0 \sin 2\pi ft$ is created by the PEM in two polarization directions along the principal axes. The expression of the light field after the PEM is given by:

$$E_x = \frac{E_{p,m}(\vec{\boldsymbol{r}})}{\sqrt{2}}, \quad E_y = \frac{E_{p,m}(\vec{\boldsymbol{r}})}{\sqrt{2}}e^{i\delta} \tag{S2.6}$$

Then, σ can be written as:

$$\sigma = E_y/E_x = e^{i\delta} = \sigma_r + i\sigma_i \tag{S2.7a}$$

$$\sigma_r = \cos\delta\,, \sigma_i = \sin\delta, \quad \frac{1}{1+|\sigma|^2} = \frac{1}{2} \tag{S2.7b}$$

The expression of $m \cdot \sigma_i \boldsymbol{J}_{(1)}^{\rho}$ **is as** follows:

$$m \cdot \sigma_i \boldsymbol{J}_{(1)}^{\rho} = m \cdot \frac{2E_0^2 \left|u_{p,m}(\rho, z)\right|^2}{\rho} \left(S_i^{xxyy} - S_i^{xyxy}\right) \sin[\delta_0 \sin(2\pi f t)]$$
$$= m \cdot C(\vec{\boldsymbol{r}}) \sin[\delta_0 \sin(2\pi f t)] \quad \text{(S2.8)}$$

where $\sin\delta = \sin[\delta_0 \sin(2\pi f t)]$ can be expanded by the integral order Bessel function:

$$\sin\delta = \sin[\delta_0 \sin(2\pi f t)] = \sum_{n=0}^{+\infty} 2J_{2n+1}(\delta_0) \sin[2(2n+1)\pi f t] \quad \text{(S2.9)}$$

Experimentally, if we extract the component of the photocurrent with frequency f via a lock-in amplifier, the component corresponds to the leading term of $m \cdot \sigma_i \boldsymbol{J}_{(1)}^{\rho}$ and can be written as:

$$m \cdot \sigma_i \boldsymbol{J}_{(1)}^{\rho} \approx m \cdot C(\vec{\boldsymbol{r}}) 2J_1(\delta_0) \sin 2\pi f t \quad \text{(S2.10)}$$

The extracted CPGE component corresponds to $m \cdot C(\vec{\boldsymbol{r}}) 2J_1(\delta_0)$. Compared with mechanical modulation with a quarter waveplate, CPGE response generation via the PEM modulation approach has an additional $2J_1(\delta_0)$ coefficient, and for $\delta_0 = \pi/2$, $2J_1(\pi/2) = 1.13365$.

S3. CPGE extraction for different polarization modulation schemes

In the last session, we only consider the difference in the CPGE extraction part because of different polarization modulation schemes; in this session, we calculate the specific expression of the extracted CPGE response under these two measurement schemes because of different CPGE readout approaches.

For PEM modulation, the CPGE response is directly extracted by the lock-in amplifier locked to the 50.14-kHz modulation frequency, and the lock-in amplifier outputs the root mean square of the CPGE response, which can be written as:

$$J_C^{PEM} = m \cdot C(\vec{\boldsymbol{r}}) 2J_1(\delta_0)/\sqrt{2} \quad \text{(S3.1)}$$

For mechanical modulation, to exclude background signals and reduce 1/f noise, the OAM beams are also chopped by a mechanical chopper in an on-off manner, and the photocurrent response is extracted by a lock-in amplifier locked to the chopping frequency f'. Under this measurement scheme, the photocurrent response to chopped light can be written as:

$$J_{mod} = \begin{cases} m \cdot \sigma_i \boldsymbol{J}_{(1)}^{\rho}, & \dfrac{k}{f'} \leq t < \dfrac{k + 1/2}{f'}, \ k \in Z \\ 0, & \dfrac{k + 1/2}{f'} \leq t < \dfrac{k+1}{f'}, \ k \in Z \end{cases} \quad \text{(S3.2)}$$

The lock-in amplifier extracts the component with frequency f' and outputs the root mean square of the photocurrent, which is given by:

$$J_{rms} = \frac{f'}{\sqrt{2}} \int_0^{1/f'} J_{mod} \cdot 2\sin(2\pi f' t)\, dt$$
$$= m \cdot C(\vec{\boldsymbol{r}}) \sin 2\theta \cdot \frac{2}{\pi}/\sqrt{2} \quad \text{(S3.3)}$$

The extracted CPGE response is given by:

$$J_C^{QWP} = m \cdot C(\vec{\boldsymbol{r}}) \cdot \frac{2}{\pi}/\sqrt{2} \quad \text{(S3.4)}$$

For mechanical and PEM modulations, the extracted CPGE response differs with different constant coefficients because of different CPGE measurement schemes. Theoretically, the extracted CPGE response with PEM modulation is $\pi J_1(\pi/2) \approx 1.78$ times that with mechanical modulation.

S4. Response time of the device

In this session, we present the characterization of the photocurrent response time of a typical MLG device used in this work. We measure the second-order DC photocurrent under excitation by a basic mode Gaussian beam with no OAM. The beam is electrically modulated at different frequencies (f) (up to 100 kHz) with the controller of the quantum cascade laser, allowing us to evaluate the device's response time, as shown in Fig. S2. The response time (τ) of the device is obtained by fitting with the following function:

$$R(f) = \frac{R_0}{\sqrt{1 + (2\pi f \tau)^2}} \quad \text{(S4.1)}$$

The results indicate that the device has a response time of $\tau = 3.42$ µs, which is mainly limited by the RC constant of the device.

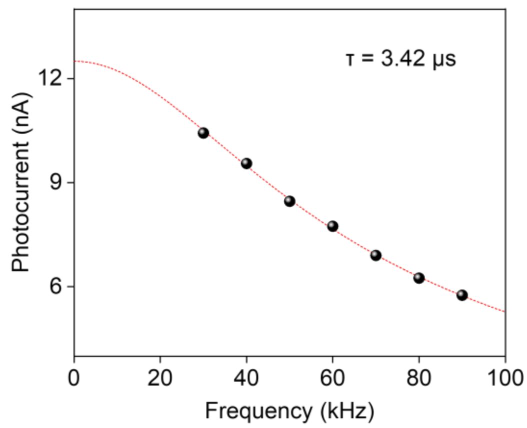

Fig. S1 Measurements of the response time for modulation by the laser controller